Image Enhancement by 2-D non-Separable Quaternionic Filter Bank-based Thresholding Neural Network

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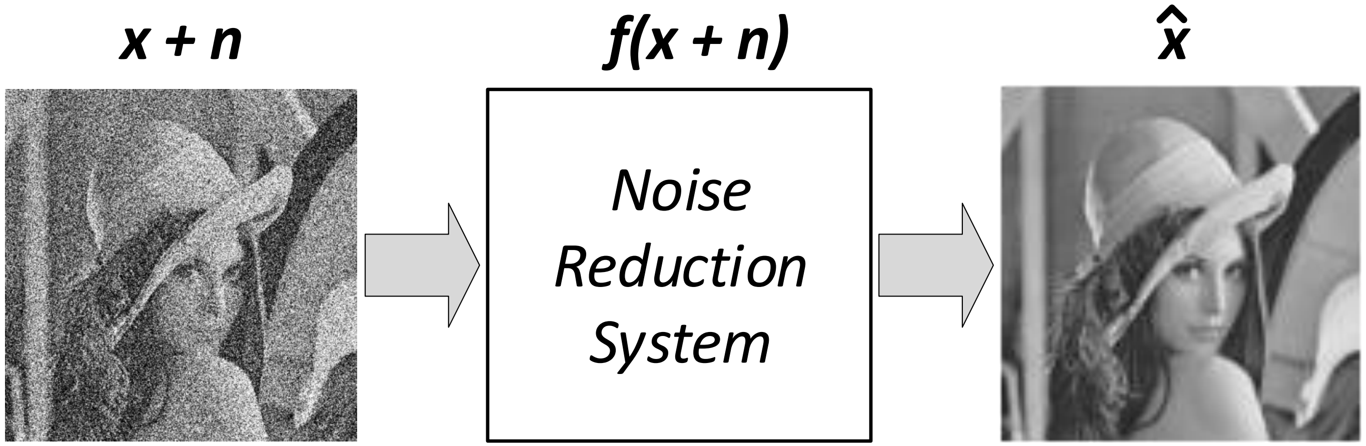
14th International Conference on Pattern Recognition and Information Processing (PRIP'2014)

Minsk, Belarus: May 21-23, 2019

NOISE REDUCTION PROBLEM

- Generic noise reduction system structure



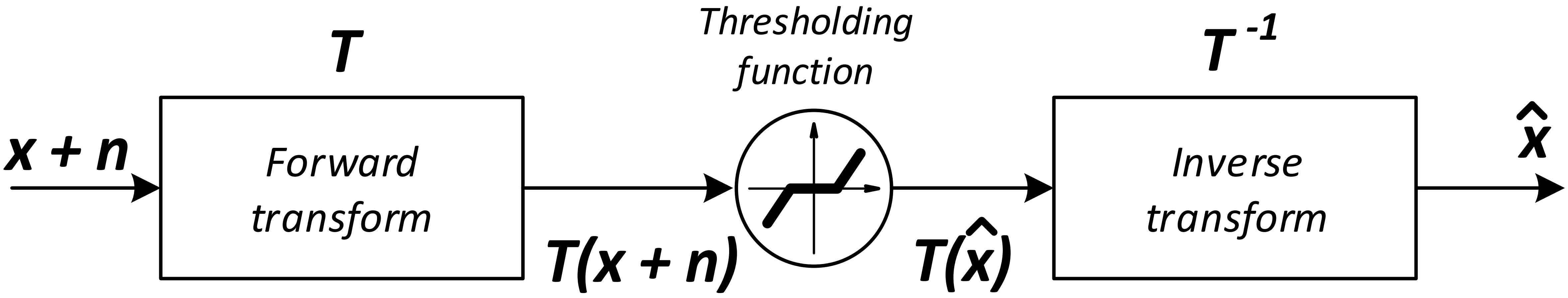
- The objective of noise reduction is to minimize…

$$E(\boldsymbol{x}, \widehat{\boldsymbol{x}}) = ||\boldsymbol{x} - \widehat{\boldsymbol{x}}||^2$$

$$\min_{\boldsymbol{x}, \widehat{\boldsymbol{x}} \in \mathbb{R}} E(\boldsymbol{x}, \widehat{\boldsymbol{x}}) = \min_{\boldsymbol{x}, \widehat{\boldsymbol{x}} \in \mathbb{R}} ||\boldsymbol{x} - \widehat{\boldsymbol{x}}||^2$$

NOISE REDUCTION BY THRESHOLDING (1/2)

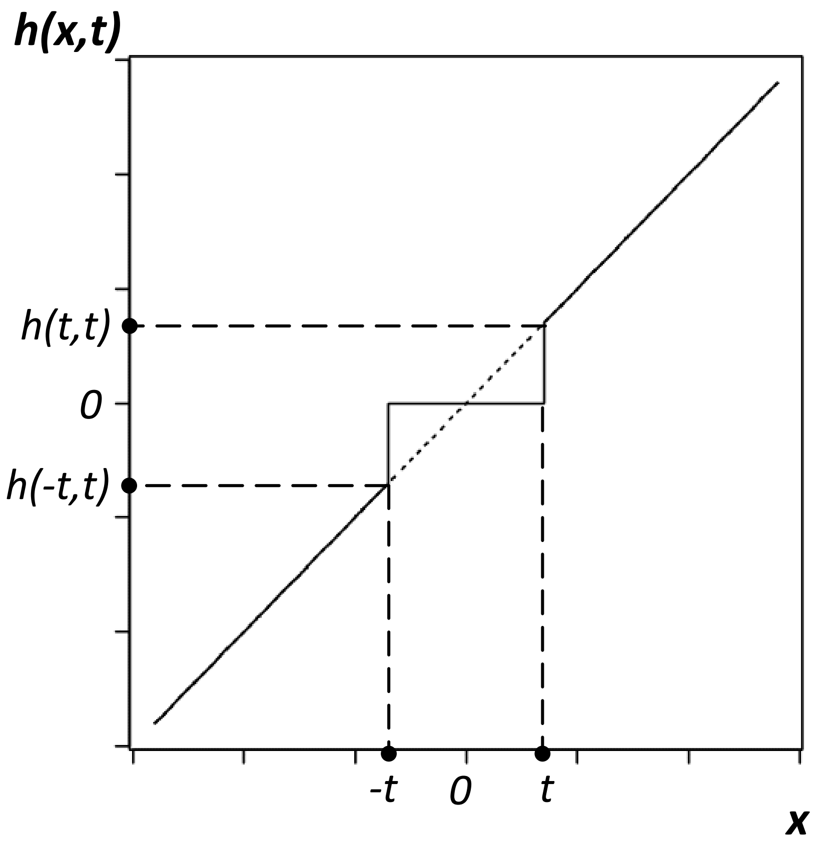
- Thresholding noise reduction approach



- Any available a priori information about the signal energy and noise distribution should be used when estimating the noise
- The useful signal can be extracted from its distorted version by processing in the transform domain with a thresholding function, which preserves the energy of the transformation coefficients that fall into the range of the distribution of the useful signal, and suppressing energy of noisy coefficients that do not fall into this area

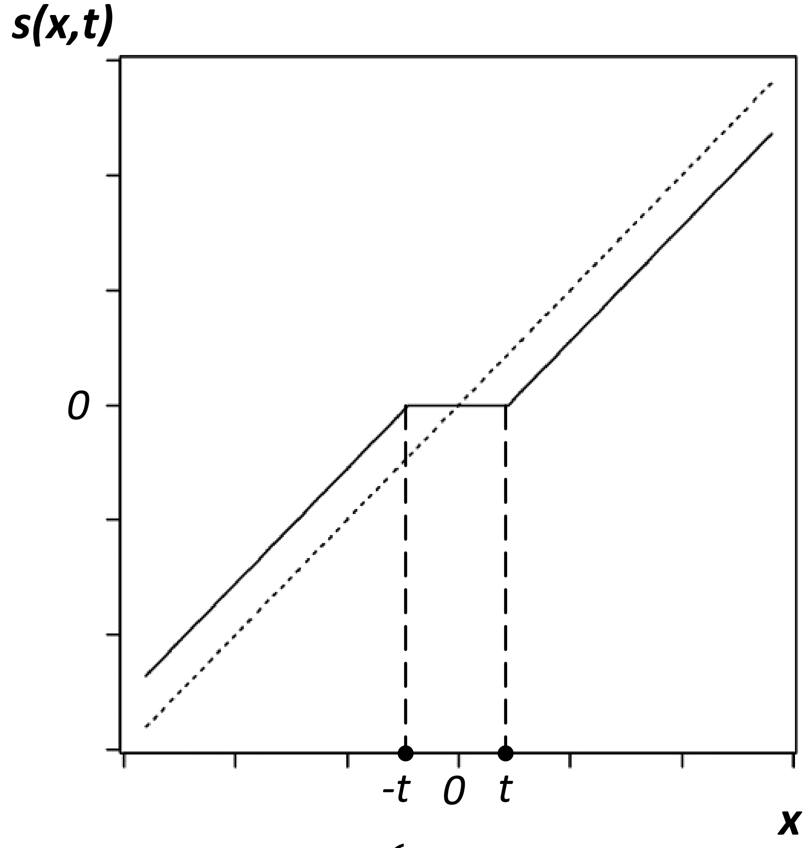
NOISE REDUCTION BY THRESHOLDING (2/2)

- Hard-Thresholding function



$$h(x,t) = \begin{cases} x, |x| > t \\ 0, |x| \leq t \end{cases}$$

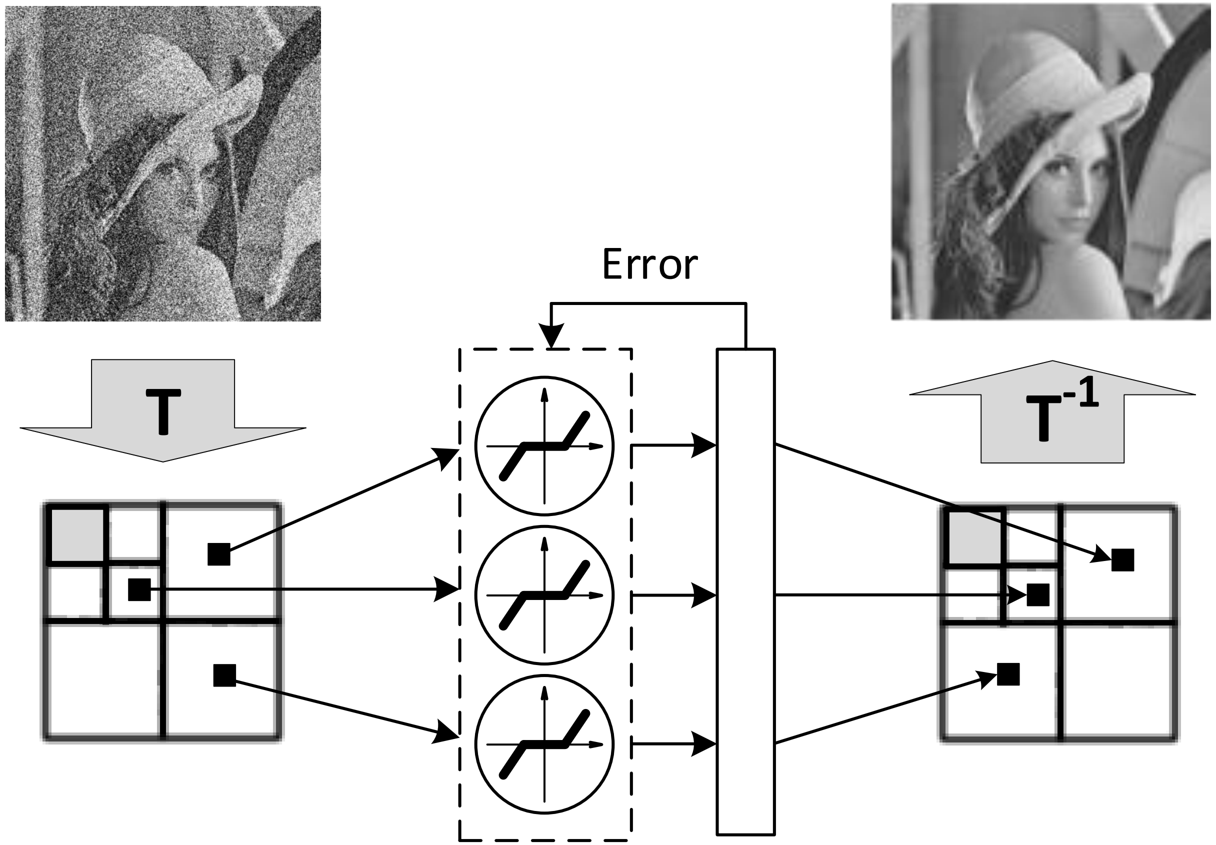
- Soft-Thresholding function



$$s(x,t) = \begin{cases} x + t, x < -t \\ x - t, x > t \\ 0, |x| \leq t \end{cases}$$

THRESHOLDING NEURAL NETWORK (1/3)

- The structure of thresholding neural network



- The term «neural network» is used here because this structure is based on the basic elements of classical neural networks such as interconnection, nonlinear activation function and it is adaptable to the input.

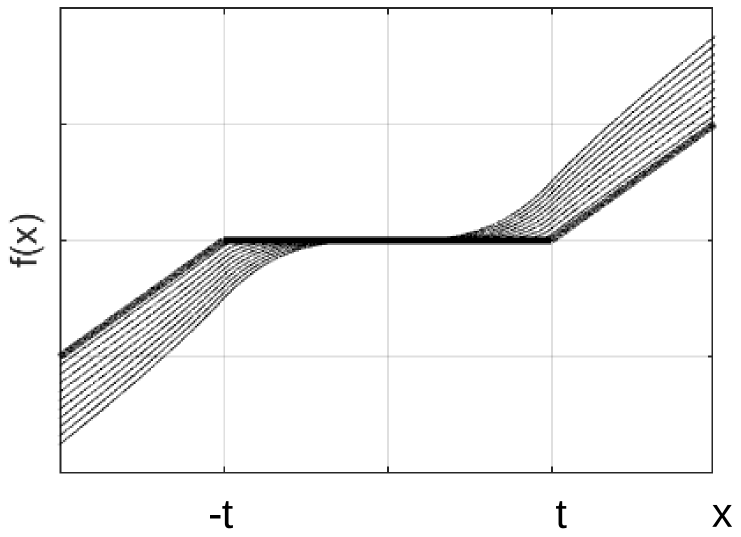
THRESHOLDING NEURAL NETWORK (2/3)

- Differentiable thresholding function[1]

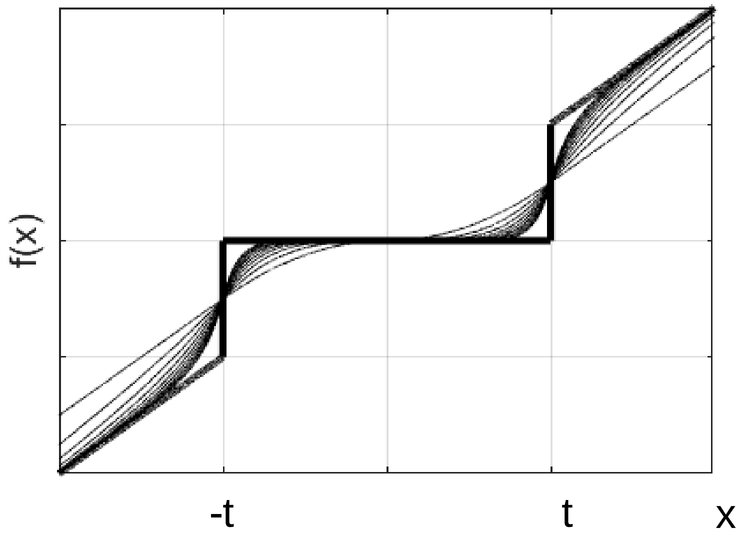
$$f(x,t,m,k)=\begin{cases} x+(k-1)t-\dfrac{kt^m}{2x^{m-1}}, & x>t \\ \dfrac{k|x|^{m+(2-k)/k}}{2t^{m+2(1-k)/k}}\, sign(x), & |x|\le t \\ x-(k-1)t+\dfrac{k(-t)^m}{2x^{m-1}}, & x<-t \end{cases}$$

- *t* – the threshold
- *m, k* – shape tuning parameters

- Function shape with respect to variation of parameter ***m***



- Function shape with respect to variation of parameter ***k***



[1] Nasri M., Nezamabadi-pour H. Image denoising in the wavelet domain using a new adaptive thresholding function // Elsevier Journal of Neurocomputing. – 2009. – V. 72. – P. 1012–1025.

- Learning algorithm

Given an initial values of *t, m, k* data *x*, maximum number of iterations *maxiter* and expected tolerance *tol*.

for *i* = 1 to maxiter **do**

1. Compute partial derivatives $\frac{\partial E(x,\hat{x})}{\partial t_i}, \frac{\partial E(x,\hat{x})}{\partial m_i}, \frac{\partial E(x,\hat{x})}{\partial k_i}$ to obtain a gradient vector.
2. Perform linear search expressed in Eq.2 for parameters $\alpha_i,\ \beta_i,\ \gamma_i$ using pre-computed gradient vector as search direction.
3. Update *t, m and k* using Eq.3.

if $E(x,\hat{x}) \leq tol$ **then**

converged

end if

end for

$$E(\boldsymbol{x},\hat{\boldsymbol{x}}) = \|\boldsymbol{x}-\hat{\boldsymbol{x}}\|^2 \tag{1}$$

$$\min_{\boldsymbol{x},\hat{\boldsymbol{x}}\in\mathbb{R}} E(\boldsymbol{x},\hat{\boldsymbol{x}}) = \min_{\boldsymbol{x},\hat{\boldsymbol{x}}\in\mathbb{R}} \|\boldsymbol{x}-\hat{\boldsymbol{x}}\|^2$$

$$\eta_i = \arg\min_{\eta_i}(E(x,\hat{x})) \tag{2}$$

$$t_{i+1} = t_i - \alpha_i \frac{\partial E(x,\hat{x})}{\partial t_i}$$

$$m_{i+1} = m_i - \beta_i \frac{\partial E(x,\hat{x})}{\partial m_i} \tag{3}$$

$$k_{i+1} = k_i - \gamma_i \frac{\partial E(x,\hat{x})}{\partial k_i}$$

TRANSFORMATION DOMAIN SELECTION

- 8-channel quaternionic critically sampled linear phase with pairwise-mirror image (PMI) symmetric frequency responses PMI LP PUFB

- Perfect reconstruction (up to scaling)
- Linear phase (LP)
- Processing as it is associated with the smoothness of the wavelet basis
- Good suitability for FPGA and VLSI implementations[1]

Polyphase transfer matrix analysis part:

$$\mathbf{E}(z) = \mathbf{G}_{N-1}(z)\mathbf{G}_{N-2}(z)\ldots\mathbf{G}_1(z)\mathbf{E}_0,$$

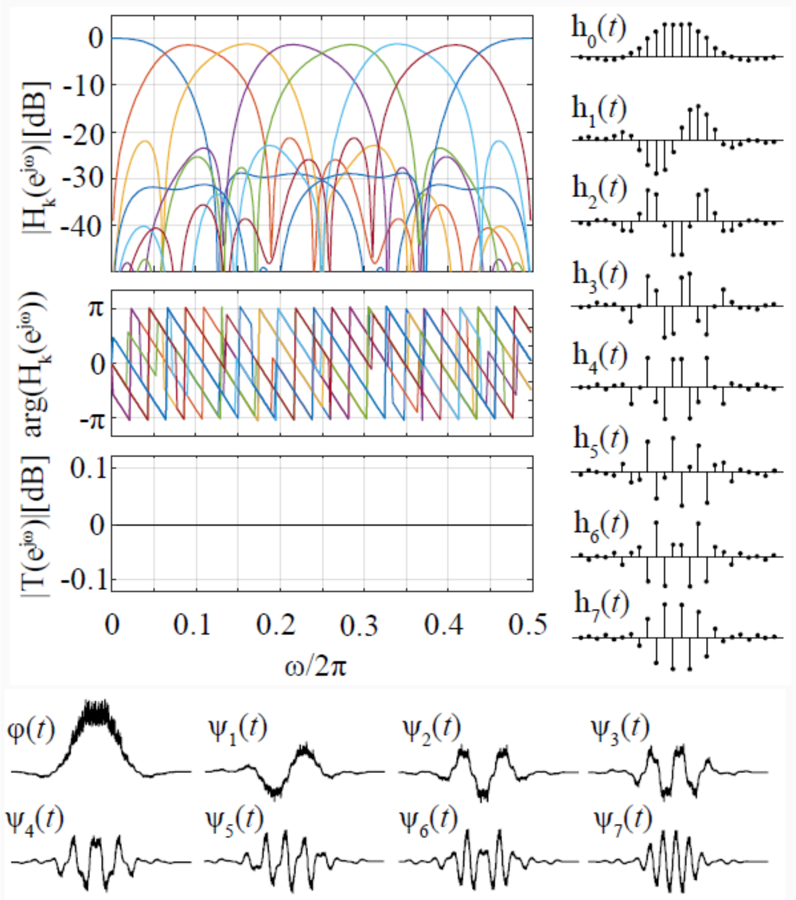
$$\mathbf{E}_0 = \frac{1}{\sqrt{2}}\mathbf{\Phi}_0 \cdot \mathbf{W} \cdot \mathrm{diag}(\mathbf{I}_{M/2}, \mathbf{J}_{M/2}),$$

$$\mathbf{G}_i(z) = \frac{1}{2}\mathbf{\Phi}_i \cdot \mathbf{W} \cdot \mathbf{\Lambda}(z) \cdot \mathbf{W},\ i = 1, \ldots, N-1,$$

$$\mathbf{W} = \begin{bmatrix} \mathbf{I}_{M/2} & \mathbf{I}_{M/2} \\ \mathbf{I}_{M/2} & -\mathbf{I}_{M/2} \end{bmatrix},$$

$$\mathbf{\Lambda}(z) = \mathrm{diag}(\mathbf{I}_{M/2}, z^{-1}\mathbf{I}_{M/2}).$$

For M=8, N = 2 (8x24 Int-Q-PUFB):



Multidimensional signals are generally non-separable, and 1-D approach does not exploit their characteristics effectively.

For shown FB $CG_{1D} = 15{,}54\ dB$

[1] Nick A. Petrovsky, Eugene V. Rybenkov, Alexander. A. Petrovsky. Structurally orthogonal finite precision FPGA implementation of block-lifting-based quaternionic paraunitary filter banks for L2L image coding // DSP - 2017

2-D Non-separable transform

- 2-D non-separable quaternionic filter bank (2-D PMI LP NS-Q-PUFB)

- When a factorization of PMI LP Q-PUFB matrix E is applied to a 2D input signal:

$$\boldsymbol{y}_{n,n}=\mathbf{E}\cdot\boldsymbol{x}_{n,n}\cdot\mathbf{E}^{\mathrm{T}}=\underbrace{\mathbf{G}_{N-1}(\mathrm{z})\cdot\ldots\cdot\mathbf{G}_{1}(\mathrm{z})\cdot\mathbf{E}_{0}}_{\text{rows}}\cdot\boldsymbol{x}_{n,n}\cdot\underbrace{\mathbf{E}_{0}^{\mathrm{T}}\cdot\mathbf{G}_{1}^{\mathrm{T}}(\mathrm{z})\cdot\ldots\cdot\mathbf{G}_{N-1}^{\mathrm{T}}(\mathrm{z})}_{\text{columns}} \quad (1)$$

- This means that the 2D implementation of $\mathbf{G}_k(\boldsymbol{z})$ is performed after $\mathbf{G}_{k-1}(\boldsymbol{z})$ that of i.e., the matrices $\mathbf{W}$, $\mathbf{\Phi_0}$ can be operated separately.
  - $\mathbf{y}_{n,n}=\ldots\cdot\mathbf{\Phi_0}\cdot\mathbf{W}\cdot diag(\mathbf{I}_{M/2},\mathbf{J}_{M/2})\cdot\mathbf{x}_{n,n}\cdot diag(\mathbf{I}_{M/2},\mathbf{J}_{M/2})^{\mathrm{T}}\cdot\mathbf{W}^{\mathrm{T}}\cdot\mathbf{\Phi_0}^{\mathrm{T}}\cdot\cdots$ (2)

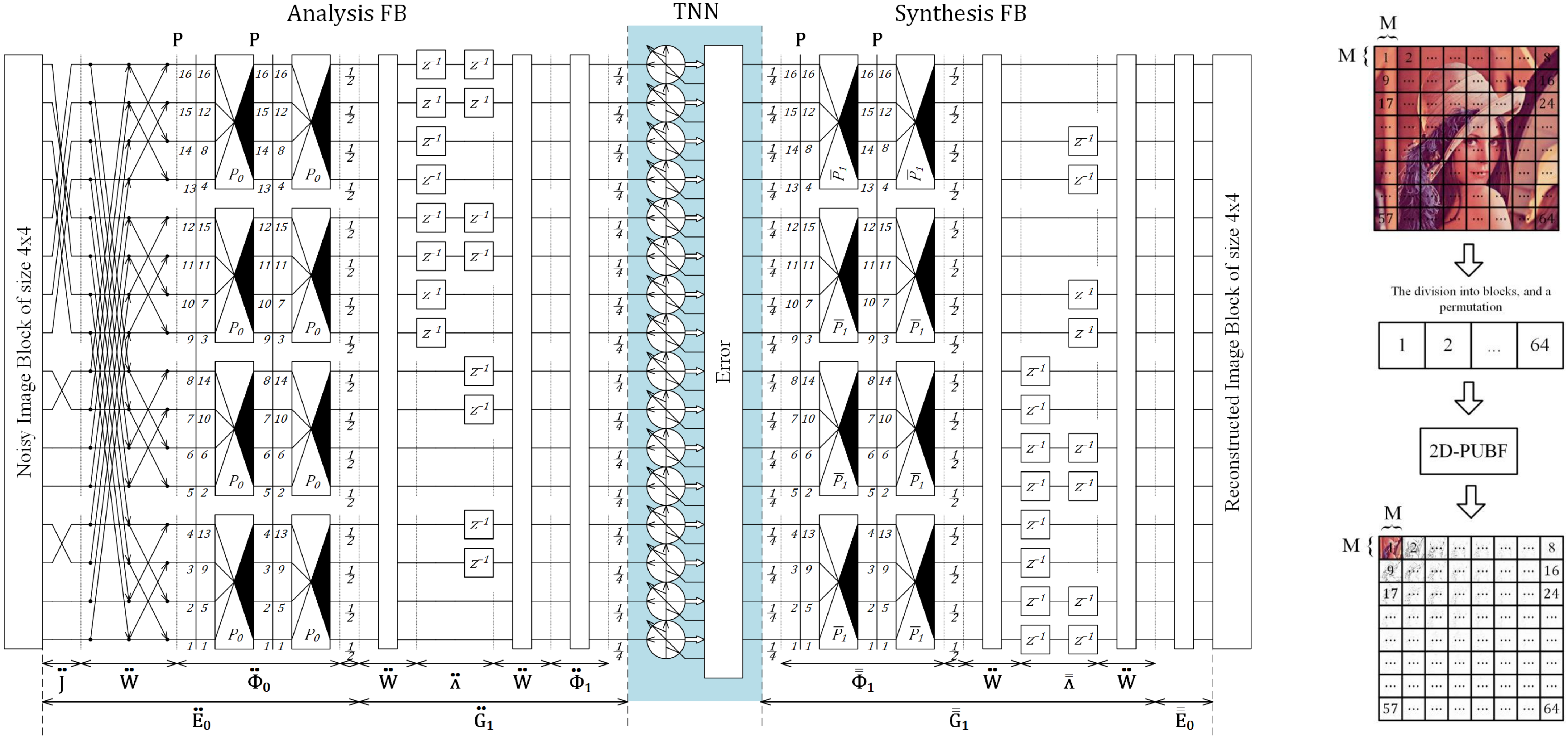
- Rewritten 2D non-separable PMI LP Q-PUFB is

$$\mathbf{y}_{n^2,1}=\ddot{\mathbf{E}}(z)\cdot\mathbf{x}_{n\cdot n,1}=\ddot{\mathbf{G}}_{N-1}(\mathrm{z})\cdot\ddot{\mathbf{G}}_{N-2}(\mathrm{z})\cdot\cdots\cdot\ddot{\mathbf{G}}_{1}(\mathrm{z})\cdot\ddot{\mathbf{E}}_{0}\cdot\mathbf{x}_{n^2,1} \quad (3)$$

- where $\ddot{}$ denotes the 2D-transformation matrix.

2D-Non Separable Q-PUFB (2D-NS-Q-PUFB)

- TNN & 2D Q-PUFB (system "16in-16out", $CG_{2D} = 17{,}12\ dB$) [1]



[1] Nick A. Petrovsky, Eugene V. Rybenkov, Alexander. A. Petrovsky. Two-dimensional non-separable quaternionic paraunitary filter banks // SPA - 2018

EXPERIMENTAL RESULTS (system “64in-64out”)

▪ Denoised test images

Original σ =10 σ =20 σ =30

▪ PSNR comparison

№	σ	Noisy	Zhang [1]	Zhang [2]	Nasri [3]	Proposed
1	10	28.16	28.74	28.09	31.67	**32.30**
	20	22.14	26.31	25.76	29.01	**29.82**
	30	18.62	25.10	24.63	27.03	**28.25**
2	10	28.16	25.32	24.58	28.42	**29.77**
	20	22.14	23.04	22.55	25.30	**25.40**
	30	18.62	22.12	21.75	**23.84**	23.56
3	10	28.16	27.05	26.42	29.97	**30.11**
	20	22.15	24.79	24.19	27.01	**27.82**
	30	18.62	23.57	23.10	25.50	**26.34**

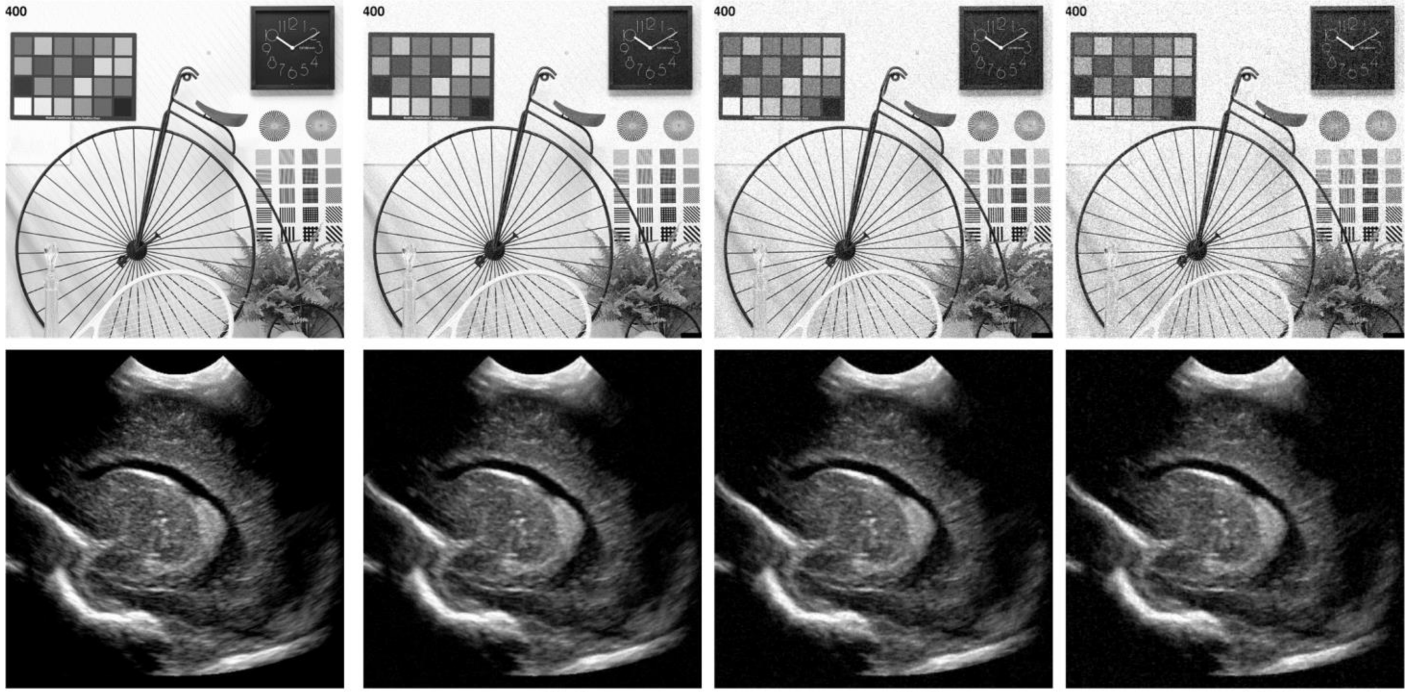
[1] X.-P. Zhang, M.D. Desai, Adaptive denoising based on SURE risk. IEEE Signal Processing Letters, 5 (10), 1998, pp. 265-267.

[2] X.-P. Zhang. Thresholding neural network for adaptive noise reduction. IEEE Transactions on Neural Networks, 12 (3), 2001, pp. 567-584.

[3] M. Nasri, H. Nezamabadi-pour. Image denoising in the wavelet domain using a new adaptive thresholding function. Elsevier Journal of Neurocomputing, 72, 2009, pp. 1012-1025.0

EXPERIMENTAL RESULTS

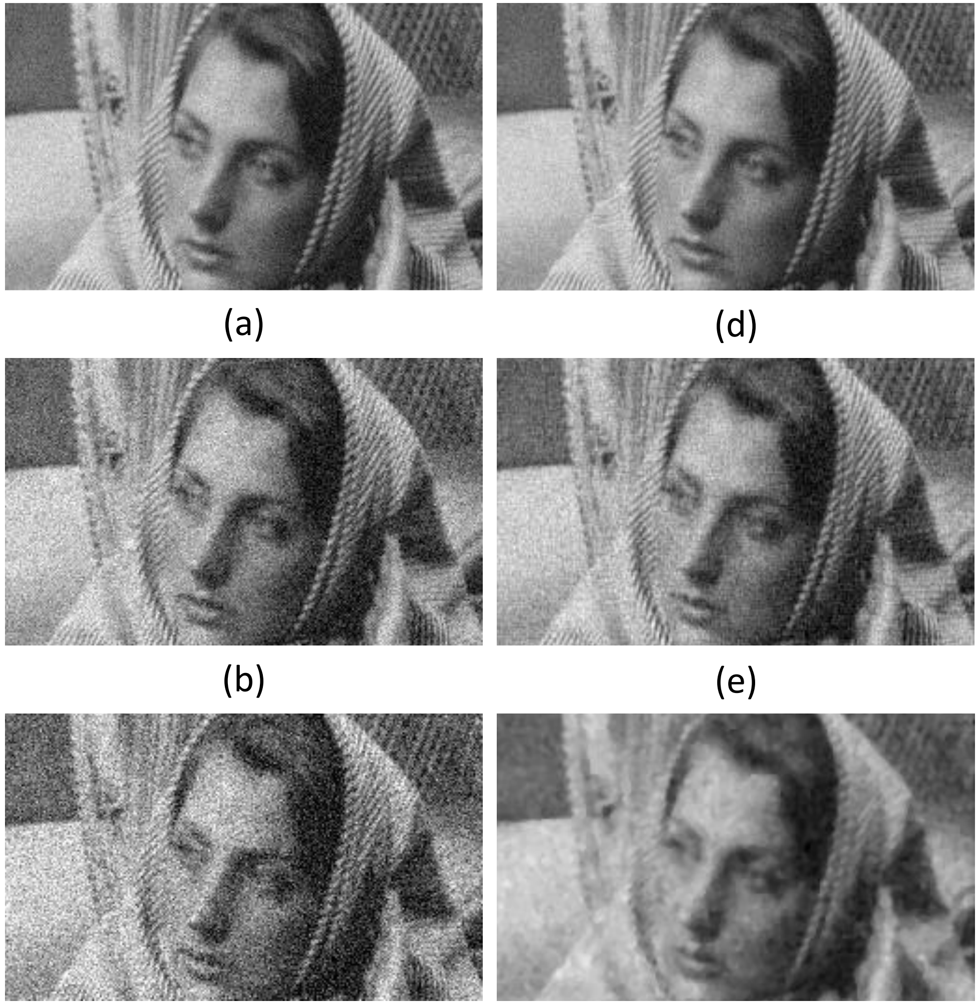
- Denoised additional test images



Original σ =10 σ =20 σ =30

Sample name	σ	Noisy image		2D-Q-PUFB-TNN	
		PSNR	SNR	PSNR	SNR
Synthetic	10	28.67	26.48	29.20	27.05
	20	22.98	20.82	24.28	22.12
	30	19.76	17.59	21.91	19.75
Ultrasound	10	29.29	18.08	34.08	22.94
	20	23.41	12.20	31.16	20.02
	30	20.05	8.84	29.45	18.31

- A detailed view of Barbara test image



(a) (d) (b) (e) (c) (f)

CONCLUSION & FUTURE WORK

- High characteristics of the multi-bands 2-D NSQ-PUFB (structure 64in-64out, CG_{2D} =17,15 dB, prototype filter bank (8x24) Q-PUFB),which forms the basis of the TNN, the results of noise editing in comparison with the approaches based on the two-channel wavelet transform in terms of PSNR are **1-1.5** dB higher.
- In further studies, it is proposed to integrate the processes of editing noise and quantizing the 2-D NS Q-PUFB coefficients in the image encoder scheme to obtain better perceptual quality in reconstructed images.