INDIVIDUAL ASSIGNMENTS

Part A. «Differential multivariable calculus »

Problem 1.

Determine the domain of the function and grapf the domain. Sketch the level curves of the function.

1.1 $z = \ln(xy)$

1.2 $z = \arcsin(xy)$

1.3 $z = \arcsin(x + y^2)$

1.4 $z = \sqrt{y^2 + x^2}$

1.5 $z = \sqrt{x^2 + y^2/4 - 1}$

1.6 $z = \arccos(3xy)$

1.7 $z = \ln(4 - x^2 - y^2)$

1.8 $z = \sqrt[4]{y^2 - x^2}$

1.9 $z = \arcsin(x^2 + y^2)$

1.10 $z = \sqrt{x^2 + y^2 - 1}$

1.11 $z = \arccos(x^2 - y^2)$

1.12 $z = \arcsin(x^2 - y^2)$

1.13 $z = \ln(x^2 + y^2 - 4)$

1.14 $z = \ln(x + y)$

1.15 $z = \sqrt{1 - x^2/4 - y}$

1.16 $z = \ln(x^2 - y^2)$

1.17 $z = \ln(\arcsin(xy))$

1.18 $z = \cos(\arccos(x^2 - y^2))$

1.19 $z = \arccos(x^2 + y^2)$

1.20 $z = \sqrt{5 - x^2 - y^2}$

1.21 $z = \ln(\frac{1}{1 - x^2 - y^2})$

1.22 $z = \sqrt{x^2 - y^2}$

1.23 $z = \arccos(\sqrt{x^2 + y^2})$

1.24 $z = (4 - x^2 - y^2)^{-1/2}$

1.25 $z = \arcsin(x^2 + y^2)$

1.26 $z = \sqrt{4 - y^2} + \frac{e^{5x+6y^2} + 1}{\sqrt[3]{1 - x^2}}$

1.27 $z = \frac{2}{y}\ln(x^2 - y^2 - 1) + \arcsin x^2$

1.28 $z = \sqrt{\ln(y^2 - x)} + \cos\frac{y^2}{\sqrt{x}}$

1.29 $z = \frac{\ln(4x^2 + y^2 - 4)}{x^2 - y^2 - 1} + e^{\frac{1}{x^2 - 1}}$

1.30 $z = \frac{\sqrt{xy}}{\ln(x^2 + y^2 - 3)} + \arccos(x^2 - y)$

Problem 2.

Find all partial derivatives of the first order of the function $U = f(x, y, z)$.

2.1 $U = x^{y^z}$

2.2 $U = (\arcsin x)^{(y^2)^{\ln z}}$

2.3 $U = (x^2)^{(y^3 - y)^{\cos z}}$

2.4 $U = (x - 5x^3)^{(\sin y)^{\cos z}}$

2.5 $U = (\cos x^2)^{(y^3 - y)^{\sin z^2}}$

2.6 $U = (x^2 - 3x)^{(3y - y^2)^{\ln z^2}}$

2.7 $U = (5x + \sin x)^{y^{\ln(z^2 - z)}}$

2.8 $U = (7\cos x - x^2)^{(\ln y)^{z^2}}$

2.9 $U = (x^2)^{(y^2)^{z^2}}$

2.10 $U = (e^{3x - x^2})^{(y^2)^{tgz}}$

2.11 $U = (\arcsin x^2)^{(\ln y)^{z^3}}$

2.12 $U = (\ln(1 - x^2))^{(y^2 + y)^{z^4}}$

2.13 $U = (x^7 - 5x^4)^{(\ln(y^2 - y))^{\cos z}}$

2.14 $U = (\ln x)^{(\cos y)^{z^2}}$

2.15 $U = (x^2)^{(\sin y)^{\ln z}}$

2.16 $U = (x^7)^{(y^6)^{z^5}}$

2.17 $U = (\ln(x^3 - 2x))^{(y^2)^{(z^3 - z)}}$

2.18 $U = (x^3 - 4x^5)^{(1 - y^2)^{(z^2 + z)}}$

2.19 $U = (x + \sin x^2)^{(\cos y)^{(z - z^2)}}$

2.20 $U = (x^5)^{(y^2)^{z^3}}$

2.21 $U = x^{(y^2)^{z^3}}$

2.22 $U = (x^2)^{(y^7)^{(z^4 - z^2)}}$

2.23 $U = (x^2)^{(\cos y^2)^{\arcsin z}}$

2.24 $U = (2x^3 - x^2)^{(e^y)^{\cos z}}$

2.25 $U = (\arcsin 3x)^{(y^2)^{tgz}}$

2.26 $U = \sin x^{\cos y^{tgz}}$

2.27 $U = tgx^{(1+x^2)^{\ln z}}$

2.28 $U = (ctg\frac{1}{x})^{\arcsin\sqrt{y}^{tgz}}$

2.29 $U = (\cos xy)^{\sin\frac{x}{z}^{ctg\frac{1}{z}}}$

2.30 $U = (\sin\frac{1}{x})^{(\cos\frac{1}{y})^{tg\frac{1}{z}}}$

Problem 3.

Find the required partial derivatives of the function $U = f(x, y)$ and verify the equation.

3.1 $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0, u = \frac{y}{x}$

3.2 $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, u = \ln(x^2 + (y+1)^2)$

3.3 $y\frac{\partial^2 u}{\partial x\partial y} = (1 + y\ln x)\frac{\partial u}{\partial x}, u = \ln(x^2 + (y+1)^2)$

3.4 $x^2\frac{\partial^2 u}{\partial x^2} + y^2\frac{\partial^2 u}{\partial y^2} = 0, u = e^{xy}$

3.5 $a^2\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2}, u = \sin^2(x - ay)$

3.6 $a^2\frac{\partial^2 u}{\partial x^2} - y^2\frac{\partial^2 u}{\partial y^2} = 0, u = y\sqrt{\frac{y}{x}}$

3.7 $a^2\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2}, u = e^{-\cos(x+ay)}$

3.8 $\frac{\partial^2 u}{\partial x\partial y} = 0, u = arctg\frac{x+y}{1-xy}$

3.9 $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, u = \ln(x^2 + y^2 + 2x + 1)$

3.10 $x^2\frac{\partial^2 u}{\partial x^2} - 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} + 2xyu = 0, u = e^{xy}$

3.11 $9\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, u = e^{-(x+3y)}\sin(x + 3y)$

3.12 $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0, u = xe^{x/y}$

3.13 $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, u = arctg\frac{y}{x}$

3.14 $\frac{\partial u}{\partial x}\cdot\frac{\partial^2 u}{\partial x\partial y} - \frac{\partial u}{\partial y}\cdot\frac{\partial^2 u}{\partial x^2} = 0, u = \ln(x + e^{-y})$

3.15 $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0, u = \ln(x^2 - y^2)$

3.16 $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0, u=-\ln\sqrt{(x-a)^2+(y-b)^2}$

3.17 $\frac{\partial^2 u}{\partial y^2}=a^2\frac{\partial^2 u}{\partial x^2}, u=A\sin mx\cos amx$

3.18 $(x-y)\frac{\partial^2 u}{\partial x\partial y}=\frac{\partial u}{\partial y}, u=\cos y+(y-x)\sin y$

3.19 $x\frac{\partial^2 u}{\partial x\partial y}-\frac{\partial u}{\partial y}, u={}^{x}\!/_{y}$

3.20 $\frac{\partial^2 u}{\partial x^2}\cdot\frac{\partial^2 u}{\partial y^2}-(\frac{\partial^2 u}{\partial x\partial y})^2=0, u=\ln(e^x+e^y)$

3.21 $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0, u=\ln\frac{1}{\sqrt{x^2+y^2}}$

3.22 $\frac{\partial^2 u}{\partial x^2}=a^2\frac{\partial^2 u}{\partial y^2}, u=\frac{y}{y^2-a^2x^2}$

3.23 $\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 u}{\partial x^2}=\frac{1}{x^2}, u=\ln(\frac{1}{x}-\frac{1}{y})$

3.24 $\frac{\partial^2 u}{\partial x^2}+2\frac{\partial^2 u}{\partial x\partial y}=0, u=arctg(2x-y)$

3.25 $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{2u}{9}, u=\sqrt[3]{ax+by}$

3.26 $9\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial y^2}, u=e^{-\cos(x+3y)}$

3.27 $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0, u=e^x(x\cos y-y\sin y)$

3.28 $\frac{\partial^2 u}{\partial x}=16\frac{\partial^2 u}{\partial y^2}, u=\frac{y}{y^2-16x^2}$

3.29 $(\frac{\partial u}{\partial x})^2+(\frac{\partial u}{\partial y})^2+(\frac{\partial u}{\partial z})^2=1, u=\sqrt{x^2+y^2+z^2}$

3.30 $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=1, u=\sqrt{x^2+y^2+z^2}$

Part B. « Multiple Integrals»

Problem 1.

Change the order of integration of each iterated integral. Do not integrate.

1.1 $$\int_{-2}^{-1} dy \int_{-\sqrt{2+y}}^{0} f(x,y)dx + \int_{-1}^{0} dy \int_{-\sqrt{-y}} f(x,y)dx.$$

1.2 $$\int_{0}^{1} dy \int_{-\sqrt{y}}^{0} f(x,y)dx + \int_{1}^{\sqrt{2}} dy \int_{-\sqrt{2-y^2}}^{0} f(x,y)dx.$$

1.3 $$\int_{0}^{1} dy \int_{0}^{y} f(x,y)dx + \int_{1}^{\sqrt{2}} dy \int_{0}^{\sqrt{2-y}} f(x,y)dx.$$

1.4 $$\int_{0}^{1} dy \int_{0}^{\sqrt{y}} f(x,y)dx + \int_{1}^{2} dy \int_{0}^{\sqrt{2-y}} f(x,y)dx.$$

1.5 $$\int_{-\sqrt{2}}^{-1} dx \int_{-\sqrt{2-x^2}}^{0} f(x,y)dy + \int_{-1}^{0} dx \int_{x}^{0} f(x,y)dy.$$

1.6 $$\int_{0}^{\sqrt{2}/2} dy \int_{0}^{\arcsin y} f(x,y)dx + \int_{\sqrt{2}/2}^{1} dy \int_{0}^{\arcsin y} f(x,y)dx.$$

1.7 $$\int_{-2}^{-1} dy \int_{0}^{\sqrt{2+y}} f(x,y)dx + \int_{-1}^{0} dy \int_{0}^{\sqrt{-y}} f(x,y)dx.$$

1.8 $$\int_{0}^{1} dy \int_{-\sqrt{y}}^{0} f(x,y)dx + \int_{1}^{e} dy \int_{-1}^{-\ln y} f(x,y)d.x$$

1.9 $$\int_{-\sqrt{2}}^{-1} dx \int_{0}^{\sqrt{2-x^2}} f(x,y)dy + \int_{-1}^{0} dx \int_{0}^{x{\wedge}2} f(x,y)dy.$$

1.10 $$\int_{-2}^{-\sqrt{3}} dx \int_{-\sqrt{4-x^2}}^{0} f(x,y)dy + \int_{-\sqrt{3}}^{0} dx \int_{\sqrt{4-x^2}}^{1} f(x,y)dy.$$

1.11 $$\int_{0}^{1} dx \int_{1-x^2}^{1} f(x,y)dy + \int_{-\sqrt{3}}^{0} dx \int_{\sqrt{4-x^2}}^{1} f(x,y)dy.$$

1.12 $$\int_{0}^{1} dy \int_{0}^{\sqrt[3]{y}} f(x,y)dx + \int_{1}^{2} dy \int_{0}^{2-y} f(x,y)dx.$$

1.13 $$\int_{0}^{\pi/4} dy \int_{0}^{\sin y} f(x,y)dx + \int_{\pi/4}^{\pi/2} dy \int_{0}^{\cos y} f(x,y)dx.$$

1.14 $$\int_{-2}^{-1} dx \int_{-(2+x)}^{0} f(x,y)dy + \int_{-1}^{0} dy \int_{3\sqrt{3}}^{0} f(x,y)dy.$$

1.15 $$\int_{0}^{1} dy \int_{0}^{\sqrt{y}} f(x,y)dx + \int_{1}^{e} dy \int_{\ln y}^{1} f(x,y)dx.$$

1.16 $\int\limits_{0}^{1} dy \int\limits_{-\sqrt{y}}^{0} f(x,y)dx + \int\limits_{1}^{2} dy \int\limits_{-\sqrt{2-y}}^{0} f(x,y)dx.$

1.17 $\int\limits_{0}^{1} dy \int\limits_{-y}^{0} f(x,y)dx + \int\limits_{1}^{\sqrt{2}} dy \int\limits_{-\sqrt{2-y^2}}^{0} f(x,y)dx.$

1.18 $\int\limits_{0}^{1} dy \int\limits_{0}^{y^2} f(x,y)dx + \int\limits_{1}^{1} dy \int\limits_{0}^{2-y} f(x,y)dx.$

1.19 $\int\limits_{0}^{\sqrt{3}} dx \int\limits_{\sqrt{4-x^2}-2}^{0} f(x,y)dy + \int\limits_{\sqrt{3}}^{2} dx \int\limits_{-\sqrt{4-x^2}}^{0} f(x,y)dy.$

1.20 $\int\limits_{-2}^{-1} dy \int\limits_{-(2+y)}^{0} f(x,y)dx + \int\limits_{-1}^{0} dy \int\limits_{\sqrt[3]{y}}^{0} f(x,y)dx.$

1.21 $\int\limits_{0}^{1} dy \int\limits_{0}^{y} f(x,y)dx + \int\limits_{1}^{e} dy \int\limits_{\ln y}^{1} f(x,y)dx.$

1.22 $\int\limits_{0}^{1} dx \int\limits_{0}^{x^2} f(x,y)dy + \int\limits_{1}^{\sqrt{2}} dx \int\limits_{0}^{\sqrt{2-x^2}} f(x,y)dy.$

1.23 $\int\limits_{0}^{\pi/4} dx \int\limits_{0}^{\sin x} f(x,y)dy + \int\limits_{\pi/4}^{\pi/2} dx \int\limits_{0}^{\cos x} f(x,y)dy.$

1.24 $\int\limits_{-\sqrt{2}}^{-1} dy \int\limits_{-\sqrt{2-y^2}}^{0} f(x,y)dx + \int\limits_{-1}^{0} dy \int\limits_{y}^{0} f(x,y)dx.$

1.25 $\int\limits_{0}^{1} dx \int\limits_{0}^{x^3} f(x,y)dy + \int\limits_{1}^{2} dx \int\limits_{0}^{2-x} f(x,y)dy.$

1.26 $\int\limits_{0}^{\sqrt{3}} dx \int\limits_{0}^{2-\sqrt{4-x^2}} f(x,y)dy + \int\limits_{\sqrt{3}}^{2} dx \int\limits_{0}^{\sqrt{4-x^2}} f(x,y)dy.$

1.27 $\int\limits_{0}^{1} dx \int\limits_{-\sqrt{x}}^{0} f(x,y)dy + \int\limits_{1}^{2} dx \int\limits_{-\sqrt{2-x}}^{0} f(x,y)dy.$

1.28 $\int\limits_{0}^{1} dx \int\limits_{0}^{x} f(x,y)dy + \int\limits_{1}^{\sqrt{2}} dx \int\limits_{0}^{\sqrt{2-x^2}} f(x,y)dy.$

1.29 $\int\limits_{0}^{1} dy \int\limits_{0}^{\$y} f(x,y)dx + \int\limits_{1}^{\$2} dy \int\limits_{0}^{\sqrt{2-y^2}} f(x,y)dx.$

1.30 $\int\limits_{0}^{1} dx \int\limits_{0}^{\sqrt{x}} f(x,y)dy + \int\limits_{1}^{2} dx \int\limits_{0}^{\sqrt{2-x}} f(x,y)dy.$

Problem 2.

Evaluate the double (a) and the triple (b) integrals:

2.1 a) $\iint\limits_D (12x^2y^2 + 16x^3y^3)dxdy;$
$D: x = 1, y = x^2, y = -\sqrt{x}.$

b) $\iiint\limits_V dxdydz$
$V: y = 10x, y = 0, x = 1, z = xy, z = 0.$

2.2 a) $\iint\limits_D (9x^2y^2 + 48x^3y^3)dxdy;$
$D: x = 1, y = \sqrt{x}, y = -x^2.$

b) $\iiint\limits_V \frac{dxdydz}{(1 + x/3 + y/4 + z/8)^4}$
$V: x/3 + y/4 + z/8 = 1, x = 0, y = 0, z = 0.$

2.3 a) $\iint\limits_D (36x^2y^2 - 96x^3y^3)dxdy;$
$D: x = 1, y = \sqrt[3]{x}, y = -x^3.$

b) $\iiint\limits_V 15(y^2 + z^2)dxdydz$
$V: z = x + y, x + y = 1, x = 0, y = 0, z = 0.$

2.4 a) $\iint\limits_D (18x^2y^2 + 32x^3y^3)dxdy;$
$D: x = 1, y =, x^3 y = -\sqrt[3]{x}.$

b) $\iiint\limits_V (3x + 4y)dxdydz$
$V: y = x, y = 0, x = 1, z = 5(x^2 + y^2), z = 0.$

2.5 a) $\iint\limits_D (27x^2y^2 + 48x^3y^3)dxdy;$
$D: x = 1, y = x^3, y = -\sqrt{x}.$

b) $\iiint\limits_V (1+2x^3)dxdydz$

$V: y=9x, y=0, x=1, z=\sqrt{xy}, z=0$

2.6 a) $\iint\limits_D (18x^2y^2+32x^3y^3)dxdy;$

$D: x=1, y=\sqrt[3]{x}, y=-x^2.$

b) $\iiint\limits_V (27+54y^3)dxdydz$

$V: y=x, y=0, x=1, z=\sqrt{xy}, z=0$

2.7 a) $\iint\limits_D (18x^2y^2+32x^3y^3)dxdy;$

$D: x=1, y=x^2, y=-\sqrt{x}.$

b) $\iiint\limits_V ydxdydz$

$V: y=15x, y=0, x=1, z=xy, z=0.$

2.8 a) $\iint\limits_D (27x^2y^2+48x^3y^3)dxdy;$

$D: x=1, y=x^2, y=-\sqrt{x}.$

b) $\iiint\limits_V \frac{dxdydz}{1+x/16+y/8+z/3}$

$V: x/16+y/8+z/3=1, x=0, y=0, z=0.$

2.9 a) $\iint\limits_D (4xy+3x^2y^2)dxdy;$

$D: x=1, y=x^2, y=-\sqrt{x}.$

b) $\iiint\limits_V (3x^2+y^2)dxdydz$

$V: z=10y, x+y=1, x=0, y=0, z=0.$

2.10 a) $\iint\limits_D (12xy+9x^2y^2)dxdy;$

$D: x=1, y=\sqrt{x}, y=-x^2.$

b) $\iiint_V (15x+30z)dxdydz$

$V: z=x^2+3y^2, z=0, y=x, y=0, x=1.$

2.11 a) $\iint_D (8xy+9x^2y^2)dxdy;$

$D: x=1, y=\sqrt[3]{x}, y=-x^2.$

b) $\iiint_V (4+8z^3)dxdydz$

$V: y=x, z=\sqrt{xy}, y=0, x=1, z=0.$

2.12 a) $\iint_D (24xy+18x^2y^2)dxdy;$

$D: x=1, y=x^3, y=-\sqrt[3]{x}.$

b) $\iiint_V (1+2x^3)dxdydz$

$V: y=36x, y=0, x=1, z=0, z=\sqrt{xy}$

2.13 a) $\iint_D (12xy+27x^2y^2)dxdy;$

$D: x=1, y=x^2, y=-\sqrt[3]{x}.$

b) $\iiint_V 21xzdxdydz$

$V: y=x, y=0, x=2, z=xy, z=0.$

2.14 a) $\iint_D (8xy+18x^2y^2)dxdy;$

$D: x=1, y=\sqrt[3]{x}, y=-x^2.$

b) $\iiint_V \frac{dxdydz}{(x/10+y/8+z/3)^6}$

$V: x/10+y/8+z/3=1, x=0, y=0, z=0.$

2.15 a) $\iint_D (4/5\,xy+9/11\,x^2y^2)dxdy;$

$D: x=1, y=x^3, y=-\sqrt{x}.$

b) $\iiint\limits_V (x^2 + 3y^2)dxdydz$

$V : z = 10x, x + y = 1, x = 0, y = 0, z = 0.$

2.16 a) $\iint\limits_D (4/5\, xy + 9x^2y^2)dxdy;$

$D : x = 1, y = \sqrt{x}, y = -x^3.$

b) $\iiint\limits_V (60y + 90z)dxdydz$

$V : y = x, z = x^2 + y^2, z = 0, x = 1, y = 0.$

2.17 a) $\iint\limits_D (4xy + 16x^3y^3)dxdy;$

$D : x = 1, y = x^2, y = -\sqrt{x}.$

b) $\iiint\limits_V (10/3\, x + 5/3)dxdydz$

$V : y = 9x, y = 0, x = 1, z = \sqrt{xy, z = 0.}$

2.18 a) $\iint\limits_D (6xy + 24x^3y^3)dxdy;$

$D : x = 1, y = \sqrt{x}, y = -x^2.$

b) $\iiint\limits_V (9 + 18z)dxdydz$

$V : y = 4x, y = 0, x = 1, z = \sqrt{xy}, z = 0.$

2.19 a) $\iint\limits_D (4xy + 16x^3y^3)dxdy;$

$D : x = 1, y = \sqrt[3]{x}, y = -x^3.$

b) $\iiint\limits_V 3y^2 dxdydz$

$V : y = 2x, z = xy, y = 0, x = 2, z = 0.$

2.20 a) $\iint\limits_D (4xy + 16x^3y^3)dxdy;$

$D : x = 1, y = x^3, y = -\sqrt[3]{x}.$

b) $\iiint\limits_V \frac{dxdydz}{(1+x/2+y/4+z/6)^4}$

$V: x/2+y/4+z/6=1, x=0, y=0, z=0.$

2.21 a) $\iint\limits_D (44xy+16x^3y^3)dxdy;$

$D: x=1, y=x^2, y=.-\sqrt{x}$

b) $\iiint\limits_V x^2dxdydz$

$V: z=10(x+3y), x+y=1, x=0, y=0, z=0.$

2.22 a) $\iint\limits_D (4xy+176x^3y^3)dxdy;$

$D: x=1, y=\sqrt[3]{x}, y=-x^2.$

b) $\iiint\limits_V (8y+12z)dxdydz$

$V: y=x, z=3x^2+2y^2, z=0, y=0, x=1.$

2.23 a) $\iint\limits_D (xy-4x^3y^3)dxdy;$

$D: x=1, y=x^3, y=-\sqrt{x}.$

b) $\iiint\limits_V 63(1+2\sqrt{y})dxdydz$

$V: y=x, y=0, x=1, z=\sqrt{xy}, z=0.$

2.24 a) $\iint\limits_D (4xy+176x^3y^3)dxdy;$

$D: x=1, y=\sqrt{x}, y=x^2.$

b) $\iiint\limits_V (x+y)dxdydz$

$V: y=x, y=0, x=1, z=30x^2+60y^2, z=0.$

2.25 a) $\iint\limits_D (6x^2y^2+25/3\, x^4y^4)dxdy;$

$D: x=1, y=x^2, y=-\sqrt{x}.$

b) $\iiint\limits_V xyzdxdydz$

$V: y = x, y = 0, x = 2, z = x, z = 0.$

2.26 a) $\iint\limits_D (9x^2y^2 + 25x^4y^4)dxdy;$

$D: x = 1, y = \sqrt{x}, y = -x^2.$

b) $\iiint\limits_V \frac{dxdydz}{(1 + x/6 + y/4 + z/16)^5}$

$V: x/6 + y/4 + z/16 = 1, x = 0, y = 0, z = 0.$

2.27 a) $\iint\limits_D (3x^2y^2 + 50/3\, x^4y^4)dxdy;$

$D: x = 1, y = \sqrt[3]{x}, y = -x^3.$

b) $\iiint\limits_V (5x + 3z/2)dxdydz$

$V: y = x, y = 0, x = 1, z = x^2 + 15y^2, z = 0.$

2.28 a) $\iint\limits_D (9x^2y^2 + 25x^4y^4)dxdy;$

$D: x = 1, y = x^3, y = -\sqrt[3]{x}.$

b) $\iiint\limits_V y^2dxdydz$

$V: z = 10(3x + y), y + x = 1, x = 0, y = 0, z = 0.$

2.29 a) $\iint\limits_D (54x^2y^2 = 150x^4y^4)dxdy;$

$D: x = 1, y = x^2, y = -\sqrt[3]{x}, (x \geq 0).$

b) $\iiint\limits_V (x^2 + 4y^2)dxdydz$

$V: z = 20(2x + y), x + y = 1, x = 0, y = 0, z = 0.$

2.30 a) $\iint\limits_D (xy - 9x^5y^5)dxdy;$

$D: x = 1, y = \sqrt[3]{x}, y = -x^2, (x \geq 0).$

b) $\iiint\limits_V x^2dxdydz$

$V: y = 3x, y = 0, x = 2, z = xy, z = 0.$

Problem 3.

Find the volume of the body bounded by surfaces

3.1 a) $y =,16\sqrt{2}y = \sqrt{2x}, z = 0, x + z = 2;$

b) $x^2 + y^2 = 2y, z = 5/4 - x^2, z = 0;$

c) $z = 2 - 12\left(x^2 + y^2\right), z = 24x + 2;$

d) $1 \le x^2 + y^2 \le 49, -\sqrt{\frac{x^2 + y^2}{35}} \le z \le \sqrt{\frac{x^2 + y^2}{3}}, -x \le y \le 0;$

3.2 a) $y = 5\sqrt{x}, y = 5x/3, z = 0, z = 5 + 5\sqrt{x}/3;$

b) $x^2 + y^2 = y, x^2 + y^2 = 4y, z = \sqrt{x^2 + y^2}, z = 0;$

c) $z = 10((x-1)^2 + y^2) + 1, z = 21 - 20x;$

d) $4 \le x^2 + y^2 \le 64, \sqrt{\frac{x^2 + y^2}{15}} \le z \le \sqrt{\frac{x^2 + y^2}{3}}, -\sqrt{3} \le y \le 0;$

3.3 a) $x^2 + y^2 = 2, y = \sqrt{2}, y = 0, z = 0, z = 15x;$

b) $x^2 + y^2 = 8x\sqrt{2}, z = x^2 + y^2 - 64, z = 0 (z \ge 0);$

c) $z = 8(x^2 + y^2) + 3, z = 16x3;$

d) $4 \le x^2 + y^2 + z^2 \le 64, \sqrt{\frac{x^2 + y^2}{3}}, -x/3 \le y \le 0;$

3.4 a) $x + y = 2, y = \sqrt{x}, z = 12y, z = 0;$

b) $x^2 + y^2 + 4x = 0, z = 8 - y^2, z = 0;$

c) $z = 2 - 22((x+1)^2 + y^2), z = -40x - 38;$

d) $4 \le x^2 + y^2 + z^2 \le 36, z \ge -\sqrt{\frac{x^2 + y^2}{63}}, y \le 0 \le -x/\sqrt{3}.$

3.5 a) $x = 20\sqrt{2y}, y = 5\sqrt{2y}, z = 0, z + y = 1/2;$

b) $x^2 + y^2 = 6x, x^2 + y^2 = 9x, z = \sqrt{x^2 + y^2}, z = 0, y = 0 (y \le 0);$

c) $z = 4 - 14(x^2 + y^2), z = 4 - 28x;$

d) $1 \le x^2 + y^2 + z^2 \le 36, z \ge \sqrt{\frac{x^2 + y^2}{99}}, -\sqrt{3} \le y \le -x\sqrt{3}.$

3.6 a) $x = 5\sqrt{y}/2, x = 5y/6, z = 0, z = 5(3 + \sqrt{y})/6;$

b) $x^2 + y^2 = 6\sqrt{2}y, z = x^2 + y^2 - 36, z = 0 (z \ge 0);$

c) $z = 28((x+1)^2 + y^2) + 3, z = 56x + 59;$

d) $25 \le x^2 + y^2 + z^2 \le 100, z \ge -\sqrt{\frac{x^2 + y^2}{99}}, x\sqrt{3} \le y \le -x\sqrt{3}.$

3.7

$a) x^2 + y^2 = 2, x = \sqrt{y}, x = 0, z = 0, z = 30y,$

$á) x^2 + y^2 = 2y, z = 9/4 - x^2, z = 0;$

$â) z = 32(x^2 + y^2) + 3, z = 3 - 64x,$

$ã) 1 \le x^2 + y^2 + z^2 \le 49, 0 \le z \le \sqrt{\frac{x^2 + y^2}{24}}, y \le -x/\sqrt{3}, y \le -x\sqrt{3}.$

3.8

$a) x^2 + y^2 = 2, x = \sqrt{y}, z = 0, z = 12x/5,$

$á) x^2 + y^2 = 2y, x^2 + y^2 = 5y, x = \sqrt{x^2 + y^2}, z = 0;$

$â) z = 4 - 6((x-1)^2 + y^2), z = 12x - 8,$

$ã) 25 \le x^2 + y^2 + z^2 \le 121, -\sqrt{\frac{x^2 + y^2}{24}} \le z \le 0.$

3.9 a) $y = 17\sqrt{2x}, y = 2\sqrt{2x}, z = 0, x + z = 1/2;$

b) $x^2 + y^2 + 2\sqrt{2y} = 0, z = x^2 + y^2 - 4, z = 0 (z \ge 0);$

c) $z = 2 - 4(x^2 + y^{2)}, z = 8x + 2;$

d) $4 \le x^2 + y^2 \le 64, \sqrt{\frac{x^2 + y^2}{35}} \le z \le \sqrt{\frac{x^2 + y^2}{3}}, x \le y \le 0;$

3.10 a) $y = \frac{5\sqrt{x}}{3}, y = \frac{5x}{9}, z = 0, z = \frac{5(3 + \sqrt{x})}{9};$

b) $x^2 + y^2 = 4x, z = 10 - y^2, z = 0;$

c) $z = 22((x-1)^2 + y^2) + 3, z = 47 - 44x;$

d) $16 \le x^2 + y^2 + z^2 \le 100, \sqrt{\frac{x^2 + y^2}{15}} \le z \le \sqrt{\frac{x^2 + y^2}{3}}, x\sqrt{3} \le y \le 0.$

3.11 a) $x^2 + y^2 = 8, y = \sqrt{2}x, y = 0, z = 15x/11;$

b) $x^2 + y^2 = 7x, x^2 + y^2 = 10x, z = \sqrt{x^2 + y^2}, z = 0, y = 0 (y \le 0);$

c) $z = 24(x^2 + y^2) + 1, z = 48x + 1;$

d) $16 \le x^2 + y^2 + z^2 \le 100, z \le \sqrt{\frac{x^2 + y^2}{3}}, -x\sqrt{3} \le y \le -x/\sqrt{3}.$

3.12 a) $x + y = 4, y = \sqrt{2x}, z = 3y, z = 0;$

b) $x^2 + y^2 = 8\sqrt{2}y, z = x^2 + y^2 - 64, z = 0 (z \ge 0);$

c) $z = 2 - 18((x+1)^2 + y^2), z = -36x - 34;$

d) $16 \le x^2 + y^2 + z^2 \le 64, z \ge \sqrt{\frac{x^2 + y^2}{63}}, -x/\sqrt{3} \le y \le -x\sqrt{3}.$

3.13 a) $x = 5\sqrt{y}/6$, $x = 5y/18$, $z = 0$, $z = \dfrac{5(3+\sqrt{y})}{18}$;

b) $x^2 + y^2 = 2y, z = -13/4, z = 0$;

c) $z = -16(x^2 + y^2) - 1, z = -322x - 1$;

d) $4 \le x^2 + y^2 + z^2 \le 49, z \ge \sqrt{\dfrac{x^2 + y^2}{99}}, y \le 0, y \le x\sqrt{3}$.

3.14 a) $x = 19\sqrt{2y}, x = 4\sqrt{2y}, z = 0, z + y = 2$;

b) $x^2 + y^2 = 3y, x^2 + y^2 = 6y, z = 0, z = \sqrt{x^2 + y^2}$;

c) $z = 30((x+1)^2 + y^2) + 1, z = 60x + 61$;

d) $36 \le x^2 + y^2 + z^2 \le 121, z \le \sqrt{\dfrac{x^2 + y^2}{99}}, y \ge x\sqrt{3}, y \ge 0$;

3.15 a) $x^2 + y^2 = 8, x = \sqrt{2y}, x = 0, z = 30y/11, z = 0$;

b) $x^2 + y^2 = 6\sqrt{2}x, z = x^2 + y^2 - 36, z = 0 (z \ge 0)$;

c) $z = 26(x^2 + y^2) - 2, z = -52x - 2$;

d) $4 \le x^2 + y^2 + z^2 \le 64, 0 \le z \le \sqrt{\dfrac{x^2 + y^2}{24}}, y \le x\sqrt{3}, y \le x/\sqrt{3}$.

3.16

$a) x + y = 4, x = \sqrt{2y}, z = 0, z = 3x/5$,

$\acute{a}) x^2 + y^2 = 2\sqrt{2y}, z = x^2 + y^2 - 4, z = 0 (z \ge 0)$;

$\hat{a}) z = -2((x-1)^2 + y^2) - 1, z = 4x - 5$,

$\tilde{a}) 36 \le x^2 + y^2 + z^2 \le 144, -\sqrt{\dfrac{x^2 + y^2}{24}} \le z \le 0, y \ge x\sqrt{3}, y \ge x/\sqrt{3}$.

3.17 a) $y = 6\sqrt{3x}, y = \sqrt{3x}, z = 0, x + z = 3$;

b) $x^2 + y^2 = 4x, z = 12 - y^2, z = 0$;

c) $z = -2(x^2 + y^2) - 1, z = 4y - 1$;

d) $9 \le x^2 + y^2 + z^2 \le 81, \sqrt{\dfrac{x^2 + y^2}{3}} \le z \le \sqrt{\dfrac{x^2 + y^2}{15}}, 0 \le y \le -x\sqrt{3}$;

3.18

$a) y = 5\sqrt{x}/6, y = 5x/18, z = 0, z = 3x/5$,

$\acute{a}) x^2 + y^2 = 2\sqrt{2y}, z = x^2 + y^2 - 4, z = 0 (z \ge 0)$;

$\hat{a}) z = -2((x-1)^2 + y^2) - 1, z = 4x - 5$,

$\tilde{a}) 36 \le x^2 + y^2 + z^2 \le 144, -\sqrt{\dfrac{x^2 + y^2}{3}} \le z \le -\sqrt{\dfrac{x^2 + y^2}{15}}, y \ge x\sqrt{3}, y \ge x/\sqrt{3}$.

3.19

$a) x^2 + y^2 = 18, y = \sqrt{3x}, y = 0, z = 0, z = 5x/11,$

$á) x^2 + y^2 = 4x\sqrt{2}, z = x^2 + y^2 - 16, z = 0 (z \geq 0);$

$â) z = 30(x^2 + y^2) + 1, z = 60y + 1,$

$ã) 25 \leq x^2 + y^2 + z^2 \leq 144, z \leq \sqrt{\frac{x^2 + y^2}{3}}, x\sqrt{3} \leq y \leq x/\sqrt{3}.$

3.20

$a) x + y = 6, y = \sqrt{3x}, z = 0, z = 4y,$

$á) x^2 + y^2 = 4y, z = 4 - x^2, z = 0;$

$â) z = -16((x+1)^2 + y^2) - 1, z = -32x - 33,$

$ã) 36 \leq x^2 + y^2 + z^2 \leq 100, z \geq \sqrt{\frac{x^2 + y^2}{63}}, x/\sqrt{3} \leq y \leq x\sqrt{3}.$

3.21 a) $z = 7\sqrt{3y}, z = 2\sqrt{3y}, z = 0, z + y = 3;$

b) $x^2 + y^2 = 4y, x^2 + y^2 = 7y, z = \sqrt{x^2 + y^2}, z = 0;$

c) $z = 2 - 18(x^2 + y^2), z = 2 - 36y;$

d) $9 \leq x^2 + y^2 + z^2 \leq 64, z \leq \sqrt{\frac{x^2 + y^2}{99}}, y \leq x\sqrt{3}, y \leq -x/\sqrt{3};$

3.22 a) $x = 5\sqrt{y}/3, x = 5y/9, z = 0, z = 5(3 + \sqrt{y})/9;$

b) $x^2 + y^2 = 4\sqrt{y2} = x^2 + y^2 - 16, z = 0 (z \geq 0);$

c) $z = 24((9x+1)^2 + y^2) + 1, z = 48x + 49;$

d) $49 \leq x^2 + y^2 + z^2 \leq 64, z \geq \sqrt{\frac{x^2 + y^2}{99}}, y \geq x/\sqrt{3}, y \leq -x/\sqrt{3};$

3.23 a) $x^2 + y^2 = 18, x = \sqrt{3y}, x = 0, z = 0, z = 10y/11;$

b) $x^2 + y^2 + 2x = 0, z = 17/4 - y^2, z = 0;$

c) $z = 22(x^2 + y^2) + 3, z = 3 - 44y;$

d) $9 \leq x^2 + y^2 + z^2 \leq 81, 0 \leq z \leq \sqrt{\frac{x^2 + y^2}{24}}, y \leq 0 y \leq x/\sqrt{3};$

3.24 a) $x + y = 6, x = \sqrt{3y}, z = 4x/5, z = 0;$

b) $x^2 + y^2 = 9x, x^2 + y^2 = 12x, z = \sqrt{x^2 + y^2}, z = 0; y = 0 (y \geq 0)$

c) $z = 2 - 4((x-1)^2 + y^2), z = 8x - 6;$

d) $49 \leq x^2 + y^2 + z^2 \leq 169, -\sqrt{\frac{x^2 + y^2}{24}} \leq z \leq 0, y \geq 0, y \geq x/\sqrt{3};$

3.25 a) $y=\sqrt{15x}, y=x\sqrt{15}, z=0, z=\sqrt{15}(1+\sqrt{x});$

b) $x^2+y^2+2\sqrt{2}x=0, z=x^2+y^2-4, z=0(z\geq 0);$

c) $z=4-6(x^2+y^2), z=12y+4;$

d) $16\leq x^2+y^2+z^2\leq 100, \sqrt{\frac{x^2+y^2}{3}}\leq z\leq\sqrt{\frac{x^2+y^2}{35}}, 0\leq y\leq x.$

3.26 a) $x^2+y^2=50, y=\sqrt{5x}, y=0, z=0, z=3x/11;$

b) $x^2+y^2=4y, z=6-x^2, z=0;$

c) $z=32((x-1)^2+y^2)+3, z=67-64x;$

d) $16\leq x^2+y^2+z^2\leq 196, \sqrt{\frac{x^2+y^2}{3}}\leq z\leq\sqrt{\frac{x^2+y^2}{15}}, 0\leq y\leq\sqrt{3x}.$

3.27 a) $y+x=8, y=\sqrt{4x}, z=3y, z=0;$

b) $x^2+y^2=10x, x^2+y^2=13x, z=\sqrt{x^2+y^2}, z=0, y=0(y\geq 0);$

c) $z=28(x^2+y^2)+3, z=56y+3;$

d) $16\leq x^2+y^2+z^2\leq 196, z\leq\sqrt{\frac{x^2+y^2}{3}}, x/\sqrt{3}\leq y\leq 0.$

3.28 a) $x=16\sqrt{2y}, x=\sqrt{2y}, z+y=2, z=0;$

b) $x^2+y^2=2\sqrt{2}x, z=x^2+y^2-4, z=0(z\geq 0);$

c) $z=4-14((x+1)^2+y^2), z=-28x-24;$

d) $16\leq x^2+y^2+z^2\leq 144, z\geq\sqrt{\frac{x^2+y^2}{63}}, 0\leq y\leq x/\sqrt{3}.$

3.29 a) $x=15\sqrt{y}, x=15y, z=0, z=15(1+\sqrt{y});$

b) $x^2+y^2=2x, z=21/4-y^2, z=0;$

c) $z=2-20(x^2+y^2), z=2-40y;$

d) $16\leq x^2+y^2+z^2\leq 81, z\geq\sqrt{\frac{x^2+y^2}{99}}, y\leq 0, y\leq -x\sqrt{3}.$

3.30 a) $x^2+y^2=50, x=\sqrt{5y}, z=0, x=0, z=6y/11;$

b) $x^2+y^2=5y, x^2+y^2=8y, z=\sqrt{x^2+y^2}, z=0;$

c) $z=8((x+1)^2+y^2)+3, z=16x+19;$

d) $64\leq x^2+y^2+z^2\leq 169, z\leq\sqrt{\frac{x^2+y^2}{99}}, y\geq 0, y\geq -x\sqrt{3}.$